

Homework №2

Predicates

1. Are the formulas equivalent: $F_1 = (\forall x)(F(x) \vee G(x))$, $F_2 = (\exists x)F(x) \rightarrow (\exists x)G(x)$?
2. Reduce to the Skolem normal form: $[(\forall x)A(x) \rightarrow (\exists y)(\forall z)B(y, z)] \rightarrow (\forall x)C(x)$.
3. Show that the reasoning is wrong:

Any who can solve this problem is a mathematician. John is a mathematician though he can not solve it. Hence, nobody can solve this problem.

4. Write the predicate "There is the only common divisor of the numbers u and v" as a logical formula of the signature $\langle N, P(x,y), Q(x,y) \rangle$, where $P(x,y)$ means "x is a divisor of y", and $Q(x,y)$ means "x is equal to y".
5. Using the resolution method prove that the formula G is a logical consequence of formulas F_i :
 $F_1 = (\forall x)(P(x) \& R(x) \rightarrow Q(x))$, $F_2 = (\exists x)(P(x) \& R(x))$, $G = (\exists x)(Q(x) \& R(x))$.
6. Using the resolution method prove that the reasoning is right:

(Lewis Carrol. Symbolic Logic. Part I. Elementary. London, 1896)

- (1) Babies are illogical;
 - (2) Nobody is despised who can manage a crocodile;
 - (3) Illogical persons are despised.
- Therefore, babies can not manage a crocodile.

Univ. "persons "; a = able to manage a crocodile ; h = babies; c = despised; d = logical.

7. Is the formula F satisfiable? Is the formula F true identically? Is the formula F false identically?
 $F = (\forall x)(\exists y)(R(x, y) \rightarrow R(y, x))$