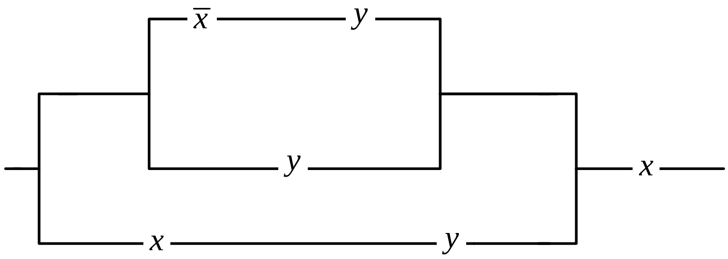
Homework №1

Propositional logic formulas

1. Are the formulas $F_1 = (\neg X \to Y) \to Y$ and $F_2 = X \to Y$ equivalent?
2. Does exist such a formula F that the formula G is identically true: $G = ((X \to Y) \to X) \to F$?

   Additionally: find all those $F(X,Y)$ that G is true, if any.
3. Is the formula G a logical consequence of formulas F_1, F_2, F_3, if
   $F_1 = X \to Y$, $F_2 = Z \to T$, $F_3 = X \vee Z$, $G = Y \vee T$? Additionally: find the number of all logical consequences of F_1, F_2.
4. To reduce the formula $\neg(X \vee Z) \wedge (X \to Y)$ to (minimal) DNF and Complete DNF.
5. If possible, simplify the relay-contact circuit



6. To prove a logical consequence $X \to Y \vee \neg T, Z \to T, X \vee Z \models Y \vee T$ with the method of resolutions.
7. Find out if the set of functions is complete $\{\sigma_1, \wedge, 0\}$, where $\sigma_1 = X \oplus Y \oplus Z$. Is that set independent?
8. Find Zhegalkin polynomial which is equivalent to the formula $X \downarrow Y \to Z$. Find corresponding minimal DNF using Carno cards.
9. With the help of PL, verify the consistency of the following reasoning

If the weather will be nice tomorrow, I'll skate or go skiing. If I'll go skiing, I'd rather go out of town, and if I'll skate, I'll stay in town. I don't want to stay in town on my day off. Therefore, if the weather will be nice tomorrow, I'll go skiing.

10. Prove that the class $\{\leftrightarrow, \vee, 0\}$ is complete reducing it to some known complete classes.
11. Find all non-equivalent formulas $F(A,B)$, those are logical consequence of formulas $A \to B$, $B \vee C$.
12. To prove that (i) $A, \neg B \vdash \neg(A \to B)$; (ii) $\vdash ((A \to B) \to A) \to A$.